A FREQUENCY TUNABLE PIEZOELECTRIC ENERGY CONVERTER BASED ON A CANTILEVER BEAM

Christoph Eichhorn, Frank Goldschmidtboeing, Peter Woias
Department of Microsystems Engineering (IMTEK), University of Freiburg, Germany
christoph.eichhorn@imtek.de

Abstract: We present a piezoelectric generator, whose resonance frequency can be tuned by applying mechanical stress to its structure. This energy harvester consists of a cantilever beam with two arms connected to the tip of the beam. The main beam contains the power generating piezoceramic material. By applying force to the arms, stress can be established in the main part of the beam which leads to a shift of its resonance frequency. First experiments with this structure indicate a high potential: Depending on the axial preload on the cantilever beam, the resonance frequency was altered from 380Hz to 292Hz. In combination with an automatic control of this force, this structure could be used to gain energy from ambient vibrations with varying frequencies. Unlike actively tuned vibration based generators [1], this energy harvester is designed in a way, that once the resonance frequency is tuned to the driving frequency, no more energy is required for resonance frequency adjustment.

Key words: Piezoelectric converter, energy harvesting, vibrations, tunable resonance frequency

1. INTRODUCTION
Transforming ambient vibrational energy to electric energy requires a good coupling between the ambient vibrations and the generator in use. The best coupling is achieved, when the ambient vibrational frequency matches exactly with the resonance frequency of the deployed generator. Also, the resonator's quality factor should be as high as possible. However, as a high quality factor comes along with a narrow resonance curve, even a tiny mismatch between the ambient frequency and the generator’s resonance frequency leads to a significant decrease in the harvested energy.

Usually, the resonance frequency of a vibrational generator is determined during the design process and, once the generator is in use, no significant further adjustment is possible. Consequently, for a proper operation, none of both frequencies is allowed to change. This constraint reduces the application field of vibrational energy harvesters dramatically, as all environments with time-dependent vibrational frequencies are excluded.

Building a broad-band energy harvester is one way to face this challenge. An example for a broad-band energy harvester is the multifrequency energy converter presented by Ferrari et al. [2]. This harvester contains various bimorph cantilevers with different natural frequencies. With broad-band harvesters, vibration energy is transduced equally in a wide frequency range and no adjustment of the harvester is necessary. Another advantage of broadband harvesters is that they can respond to different frequencies at the same time. But in environments with one dominant vibrational frequency, broadband harvesters suffer from their poor quality factor. For environmental conditions with one dominant vibrational frequency which slowly variates over time (like for example a car tire), a generator with a high quality-factor and an adjustable resonance frequency like the generator presented by Leland et al. [3] is a better solution. Their generator consists of a piezoelectric bimorph, clamped in a bench vise which applies mechanical preload to tune the resonance frequency. In our work, we pursue a similar approach with the difference that our resonator is a cantilever beam which is only clamped on one side. This design makes a bench vise with the generator’s dimensions dispensable.

2. DESIGN
The energy harvester presented in this work consists of a piezo-polymer-composite cantilever beam [4] with additional arms to apply an axial force to the tip of the beam (Fig.1). The arms connect the tip of the beam with two wings, situated on both sides of the base to which the beam is clamped. The task of the wings is to receive the force and forward it to the arms, which in turn apply the load to the tip of the beam. For stabil-
ity reasons, the wings are connected to the base of the generator. Near its base, the beam is made of a piezo-polymer-composite up to the middle of the beam. The other half up to the tip of the beam consists only of polymer material.

Fig. 1: generator with arms a) upper side with notches in the wings b) bottom side without notches

3. THEORY
To predict the behavior of our generator, we neglect the arms and assume that the beam is homogeneous. Homogeneous beams under axial load are very well described by Bokaian [5] and Shaker [6]. For small deflections, the equation of motion for a beam with an applied axial force \( P \) is:

\[
\frac{\partial^4 V(x,t)}{\partial x^4} + \frac{P}{EI} \frac{\partial^2 V(x,t)}{\partial x^2} + \frac{\rho}{EI} \frac{\partial^2 V(x,t)}{\partial t^2} = 0 \tag{1}
\]

where \( V(x,t) \) is the deflection of the beam at position \( x \) and time \( t \). \( E \) is the Young’s modulus and \( I \) the area moment of inertia of the beam cross section. \( \rho \) is the mass per length. Assuming that the beam performs harmonic oscillations with a circular frequency \( \omega \), the expression for transversal displacement can be written as:

\[
V(x,t) = v(x) \sin(\omega t) \tag{2}
\]

Assumption 2 makes equation 1 time independent:

\[
\frac{\partial^4 v(x)}{\partial x^4} + \frac{P}{EI} \frac{\partial^2 v(x)}{\partial x^2} - \frac{\rho \omega^2}{EI} v(x) = 0 \tag{3}
\]

\( v(x) \) now describes the oscillation amplitude of the beam at position \( x \). The solution to this differential equation is:

\[
v(x) = A \cosh(\alpha_1 x) + B \sinh(\alpha_1 x) \]
\[+ C \cos(\alpha_2 x) + D \sin(\alpha_2 x) \tag{4}
\]

with:

\[
\alpha_1 = \sqrt{-\frac{P}{2EI} + \sqrt{\left(\frac{P}{2EI}\right)^2 + \frac{\rho \omega^2}{EI}}} \tag{5}
\]
\[
\alpha_2 = \sqrt{-\frac{P}{2EI} + \sqrt{\left(\frac{P}{2EI}\right)^2 + \frac{\rho \omega^2}{EI}}} \tag{6}
\]

The constants A,B,C and D in equation 4 can be eliminated by considering the boundary conditions. A beam which is clamped on one side and free on the other end has the following boundary conditions:

\[
v(0) = 0 \tag{7}
\]
\[
v'(0) = 0 \tag{8}
\]
\[
v''(l) = 0 \tag{9}
\]
\[
v'''(l) + \frac{P}{EI} v'(l) = 0 \tag{10}
\]

With these boundary conditions and equation 4, a relation between the resonance frequency of the beam and the applied force can be established:

\[
2 \frac{\rho \omega^2}{EI} + \left(2 \frac{\rho \omega^2}{EI} + \frac{P}{E^2 I} \cosh(\alpha_1 l) \cos(\alpha_2 l) \right)
= \sqrt{\frac{\rho \omega^2}{EI} P \sinh(\alpha_1 l) \sin(\alpha_2 l)} \tag{11}
\]

Using adequate values for the parameters \( \rho \), \( E \), \( I \), \( l \), \( b \) and \( h \) (table 1), equation 11 can be solved numerically for the fundamental frequency. The relation between the applied axial force \( P \) and the resonance frequency of the beam is shown in figure 2. The fundamental frequency of the beam decreases when force is applied. The absolute value of the slope increases with the force, which means, that the same amount of additional force can have a bigger effect if the beam is prestressed. The fundamental frequency theoretically drops to zero, when the critical buckling point is approached.

Table 1: Values employed for the theoretical curve

| \( \rho \) | weight per length | \( 1600 \, \text{kg/m}^3 \) b h |
| \( E \) | Young’s modulus | \( 6 \, \text{GPa} \) |
| \( l \) | length of the beam | \( 20 \, \text{mm} \) |
| \( b \) | width of the beam | \( 5 \, \text{mm} \) |
| \( h \) | thickness of the beam | \( 0.44 \, \text{mm} \) |
4. EXPERIMENTAL SETUP

In this first study, the tuning force was applied by an aluminum frame (see figures 3 and 4) in combination with a screw and a steel spring. The axial load depends linearly on the deflection of the spring, which in turn is proportional to the number of revolutions of the screw. The spring pushes the whole generator base against two blocks whose counterpressure generates the prestress in the arms and the stabilizing wings.

The generator is clamped on the aluminum frame, which is fixed to a shaker performing sinusoidal oscillations. The resonance frequency of the beam was measured versus the applied force, and resonance curves were taken for different axial preloads. The acceleration amplitude of the shaker was set to approximately 6.5g.

5. RESULTS

As shown in figure 5, the whole resonance curve of our cantilever beam is shifted under preload. Like in the theoretical curve (figure 2) the resonance frequency declines, and the absolute value of the slope increases when more force is applied (figure 7). As the theory does not consider the piezoelectric part of the beam the comparison is only qualitatively.

The frequency tuning range of this generator is limited by the stability of the material in use, as the structure can handle only a certain amount of force below the fracture limit. The beam’s resonance frequency shift also comes along with a non negligible decrease of the resonator’s quality factor (Fig. 6) or, in other words, an increasing damping. As the arms used to apply force to...
with tunable resonance frequency, some energy will be required for a control unit which determines the driving frequency and decides whether further adjustment is necessary or not, and a finite amount of energy will be needed for the adjustment process. The generator will have to be dimensioned in a way that enough energy is harvested for both the control unit and the main application to be powered. The power consumption for frequency adjustment will of course also depend on the steadiness of the environmental vibration frequency.

Mechanical prestress applied by a screw can also be of use in a harvester with automatic frequency control. A piezoelectric actuator will never reach the adjustment travel of a screw. It would therefore make sense to combine both adjustment systems. The screw might be used to tune the cantilever beam to the frequency region of interest while the piezoeactuator does the fine-tuning. Mechanical prestress could also be used to enlarge the frequency tuning range of a piezoelectric actuator. As can be seen in figure 7, the same amount of additional force leads to a larger resonance frequency shift, if the beam is prestressed.

References

6. DISCUSSION AND OUTLOOK
This work shows the feasibility of frequency tunable cantilever beams for micro energy harvesting, which so far are only described in theory [7]. The next step will be to apply the mechanical stress with piezoelectric actuators which is essential for an automatic resonance frequency adjustment. In a future micro energy harvester...